The profile of the horizon: computation and use in sundials

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The profile of the horizon, obtained through experimental measurements or computed from altitude data available on the web, can be used to compute the lighting conditions of the dial and to draw the line of the horizon and the lines of the remaining hours of light.

Introduction

I suggested in a previous article [1] how to evaluate the lighting conditions of a place in order to better identify the best location where to install a new sundial.
In that article the following elements where taken into account:

- the latitude of the place
- the declination and the inclination of the dial plane
- the presence of balconies, roofs etc.
- the presence of nearby buildings.

Actually there is one more element that can heavily influence the hours of light of the dial: the profile of the horizon. If a sundial is built in a valley between the mountains, all the theoretical computations of the time of sunrise and sunset have no meaning and can be in error even of hours.

This same error applies to the horizon line that is often traced on a vertical dial. This is a horizontal straight line drawn through the base of the ortho-style and it represents the position of the sun when its height is 0 degrees above the horizon. Because of the presence of mountains this line is just theoretical and it is not correct: sunrise will happen later and sunset will happen earlier than what represented on the dial.

These both problems (to evaluate the lighting conditions of the dial and to correctly trace the horizon line) require the knowledge of the azimuth/height profile of the horizon. This information can be obtained through experimental measurements by means of a theodolite. However this instrument is not easily available to everyone, moreover it is not always possible to go to the dial place to take measurements.

A numerical method is here presented that allows to obtain, with an acceptable level of accuracy, the profile of the horizon for whichever place in the world. This method makes use of an altimetry database that is freely accessible on the web[1].

It is then shown how to use this profile in order to obtain the lighting conditions of the dial, to draw the true horizon line[2] and also to add a new type of hour lines to the dial.

[1] This same method was presented in [2] where it was used in addition to experimental measurements.
[2] One experimental method to draw the horizon line directly on the wall was shown in [3].

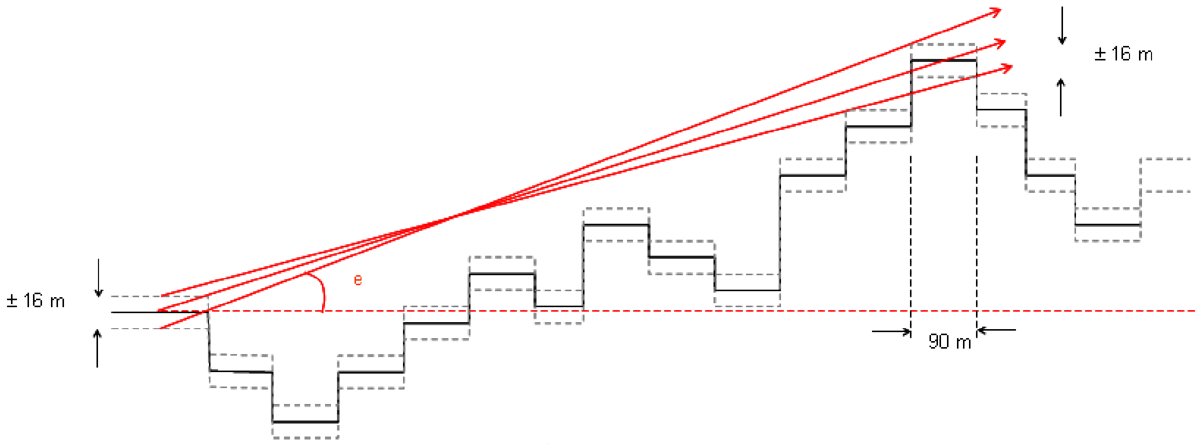

Fig. 1 – Evaluation of error in the height *e*

The DEM database

One of the missions of the space shuttle (the Shuttle Radar Topography Mission - SRTM) was used to collect elevation measurements for all the world countries between -60 and + 60 degrees of latitude. The resulting map, named Digital Elevation Model - DEM, is available from the CGIAR-CSI web site (http://srtm.csi.cgiar.org/).

The horizontal resolution is 3 seconds of arc that correspond to about 90 m (98.4 yards) at the equator. The declared vertical accuracy is ± 16 m (52.5 feet).

Data are available in 5° x 5° files having an approximate size of 150 MB (40 MB compressed). Five degrees in latitude correspond to about 540 Km (335.5 miles) while five degrees in longitude depend on the latitude and vary between 540 Km at the equator and 270 Km (167.8 miles) at ±60° of latitude.

Data are available in two different formats. In this work the .asc (arc-formatted ASCII) format was used.

Computation of the horizon profile

By means of the elevation data it is possible to compute the horizon profile as seen from the observer positioned in a known place by applying the following procedure.
Starting from the observer place proceed along a constant azimuth straight line and evaluate the height for each of the encountered points: the largest found height value is the height of the horizon for that value of azimuth. If this procedure is repeated at a constant azimuth step, the azimuth/height horizon profile is obtained.

Both the error contained in every altitude value and the resolution of 90 m limit the accuracy of the result. The precision that can be obtained when computing the height of the horizon depends on (fig. 1):

- error in altitude of the observer place (± 16 m)
- error in altitude of the horizon point (± 16 m)
- error in the evaluation of the distance of the horizon point (90 m)
- distance *d* of the horizon point

and it is shown in fig. 2.

It is evident from the graph (and it is also easy to understand) that the error is inversely proportional to the distance *d* of the horizon: at 1 Km of distance the error is about ± 4° , at 2 Km it is ± 2°.

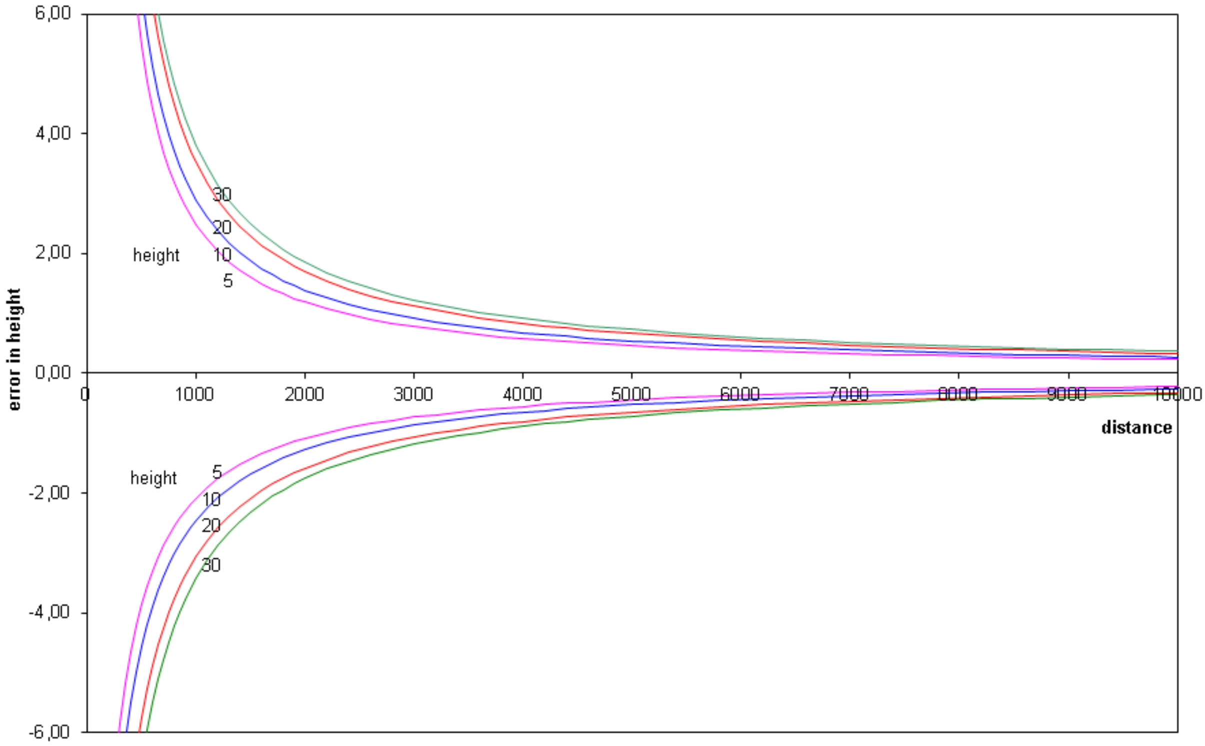

Fig. 2 – Maximum error in the evaluation of height

Resolution in azimuth depends on:

- horizontal resolution (90 m)
- distance *d* of the horizon point

and it is shown in fig. 3.

Again as expected the obtained result is better the greater the distance of the horizon is.

Results and comparisons

Some friends kindly gave me the measurements they made with a theodolite in several places in the north of Italy. In the following graphs their measurements are compared to the results obtained from DEM data with the method described above.

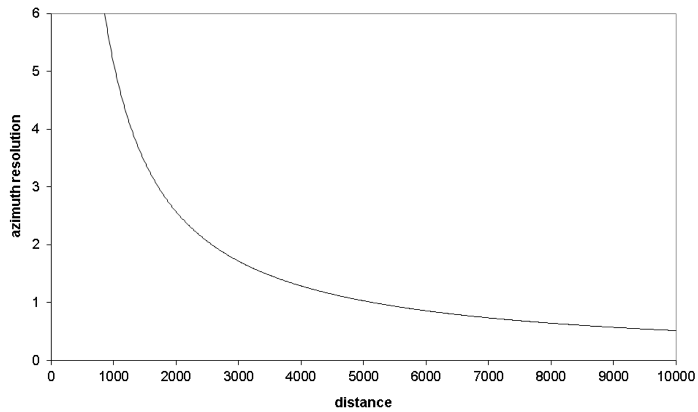

Fig. 3 – Azimuth resolution

Fig. 4 shows the comparison of the results obtained from DEM data with the measurements used by Ghia and Tasselli in [2]. The error in is less than 1°. The maximum error is mainly found around the profile peaks: this is a characteristic of DEM data that is remarked in the literature.

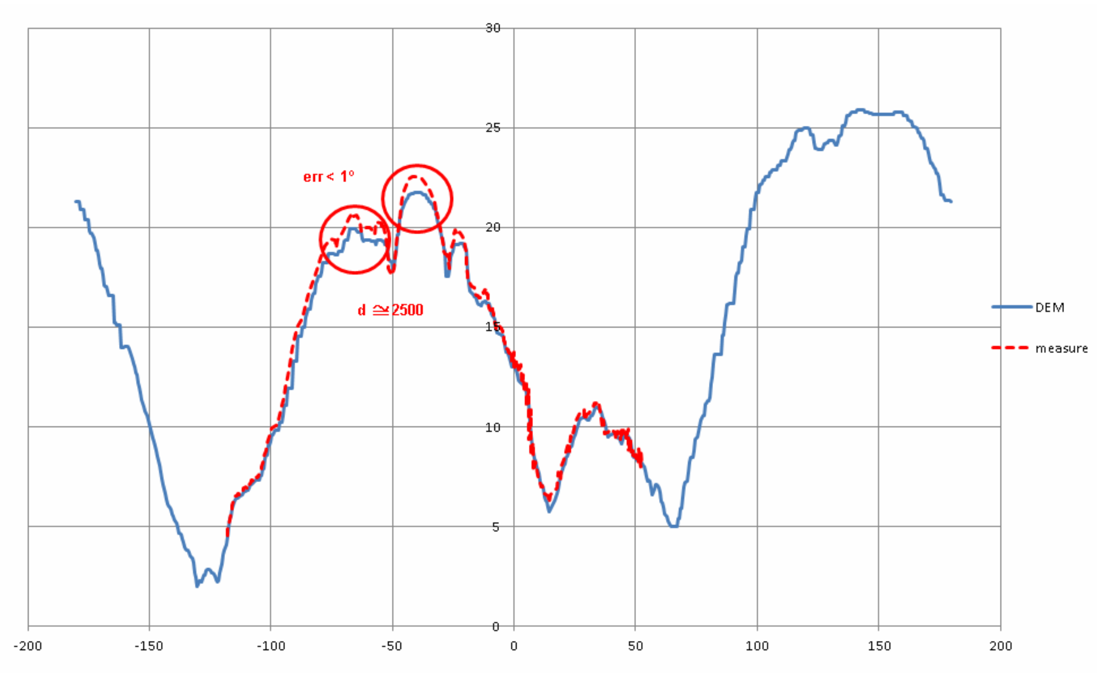

Fig. 4 – Comparison with data for Monclassico (measurements by L. Ghia and T. Tasselli)

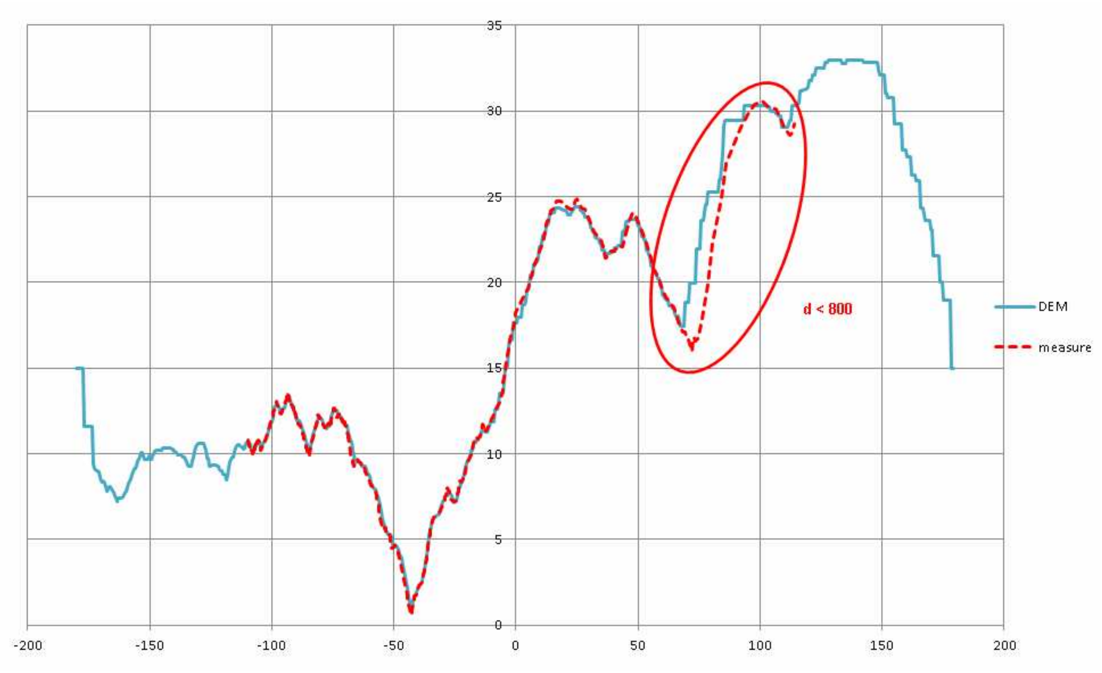

Fig. 5 – Comparison with data for Lo Pian (measurements by L. Ghia)

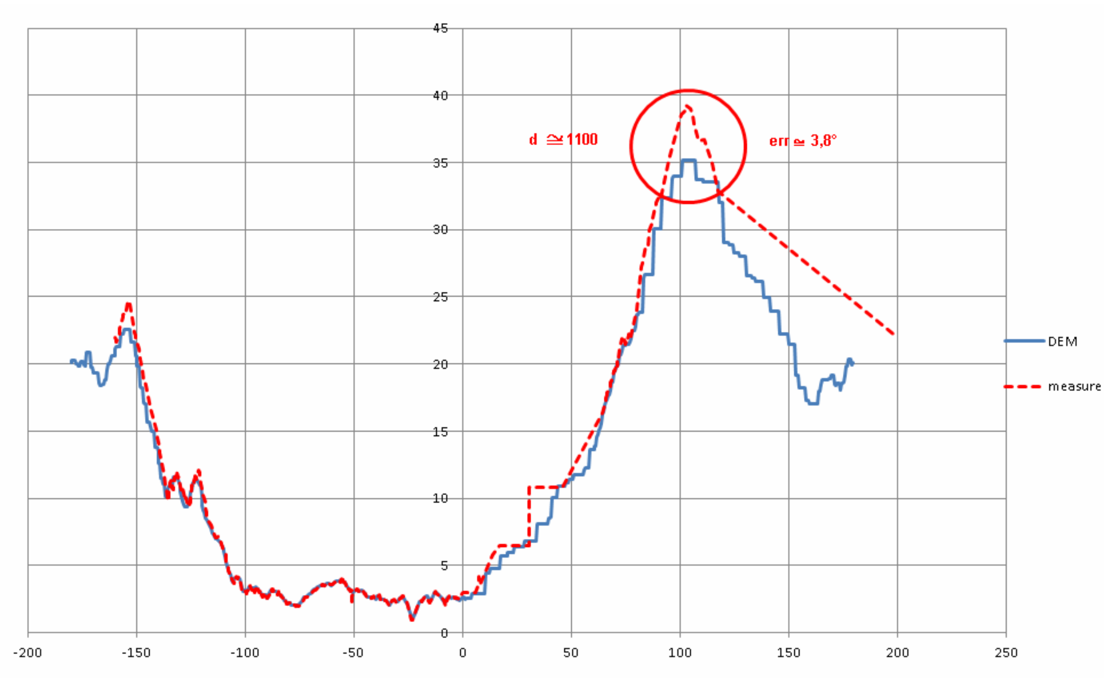

Fig. 6 – Comparison with data for Volpez (measurements by G. De Donà)

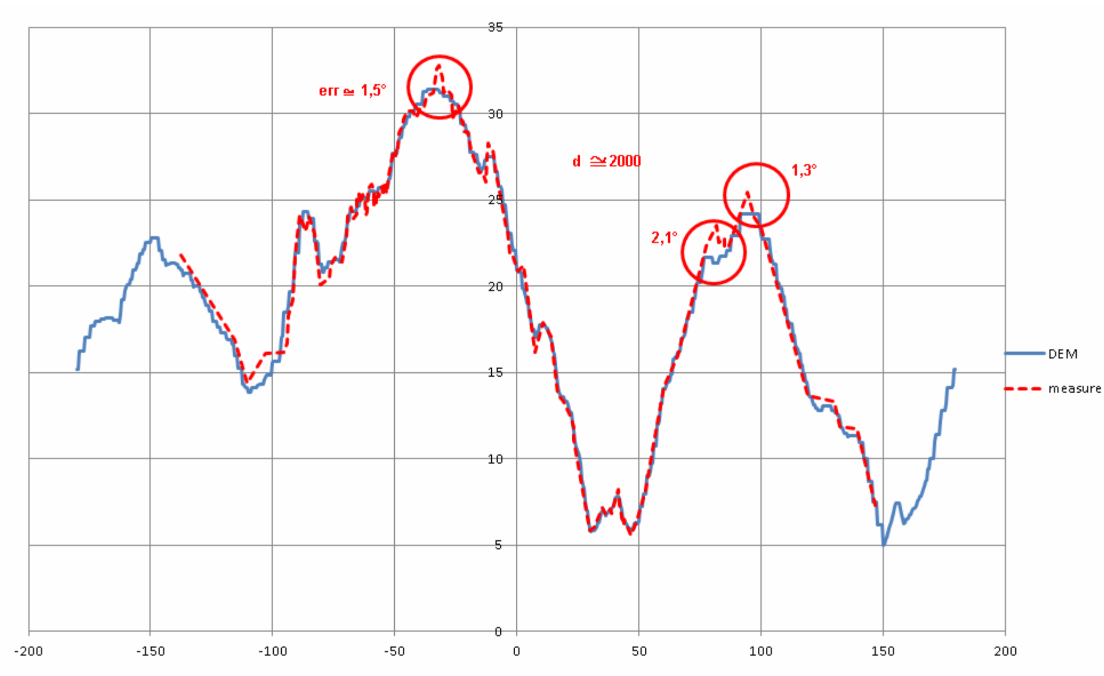

Fig. 7 – Comparison with data for Alleghe (measurements by G. De Donà)

Fig. 5 contains measurements taken by L. Ghia. Error here is as high as 4° but it should be noted that the distance of the horizon in these points is less than 1000 m and so the results of fig. 2 and 3 are confirmed.

Fig. 6 shows the comparison with measurements by De Donà. The maximum error is smaller than 4° and it is again related to peaks that are about 1000 m distant. Low azimuth resolution areas are also visible in correspondence of near points and in accordance to the results of fig. 3.

Results in fig. 7 are again by De Donà. Maximum errors are again concentrated around peaks that are under estimated.

The computation with Orologi Solari

Starting from rev. 25.0 the program Orologi Solari includes the possibility to compute the profile of the horizon for every place in the world where DEM data are available.

This new tool is run with the menu command "Tools" → "Evaluate skyline profile".
In the dialog window (fig. 8) click "load a DEM file": the DEM file is read and loaded into the program. Altimetry data are then shown in the map with arbitrary colors.

Note that if dial coordinates are near the border between different 5° x 5° tiles, the upload process needs to be repeated for each different DEM file.

Clicking the "compute the profile" button the evaluation of the horizon profile will then start.
The dialog window contains two parameters:

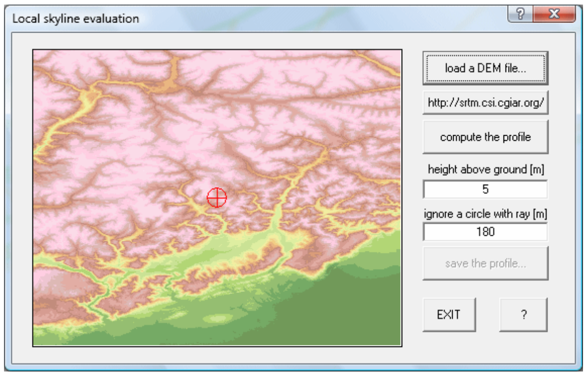

Fig. 8 – Computation of the horizon profile

- height of the dial above the ground
- radius of the circle around the dial that will be ignored when computing the profile (in order to exclude from the computation the nearest points whose uncertainty could have a strong influence on the result).

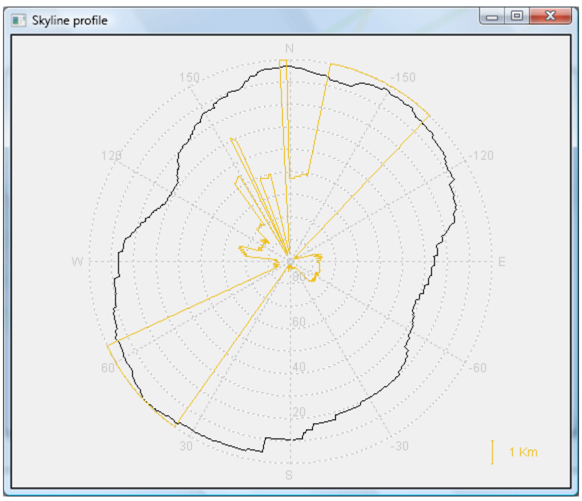

Fig. 9 – Computed profile of the horizon

Resulting profile is then shown in a polar scale graph (fig. 9) together with the distance of the horizon from the dial place.
Results can then be saved to a .ele file with the following format:

46.32416667	*= Latitude*	
-12.09611111	*= Longitude*	
720	*= NPoints (min 2, max 3600)*	
-180.000000	*3.024920*	*3706.500508*
-179.500000	*3.270936*	*3707.052511*
-179.000000	*3.270936*	*3707.052511*
-178.500000	*3.623392*	*3708.708029*
……………	…………	………………

The first two lines contain the decimal value of the geographic coordinates and the third line contains the number of points of the profile (minimum is 2, maximum is 3600).

The following lines contain the values of:

- azimuth (with respect to south, positive to west)
- horizon height angle
- distance of the horizon from the dial (in meters)

where the distance value is saved for documentation only and it is not used later.

Of course the .ele file can be prepared starting from experimental results and all the features of the program, as explained in the following paragraphs, can still be used.

How to use the profile of the horizon

In addition to the methods described in [1] for the evaluation of the lighting conditions of the dial, the information contained in the .ele file, both coming from a computation or from experimental measurements, can be added to the lighting graph.

The .ele file must first be loaded into the OS program by means of the dialog shown in fig. 10.

If now the lighting diagram of the dial is displayed, the profile of the horizon is shown in an azimuth/height diagram together with the hour and declination lines (fig. 11) and added to the effect of roofs, balconies and buildings (if defined in the program).

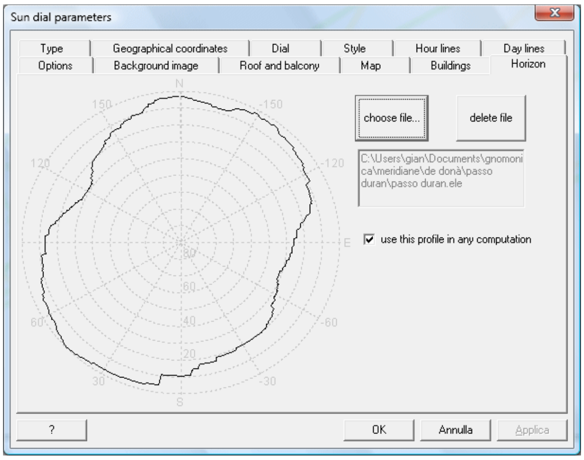

Fig. 10 – Loading the .ele file in OS

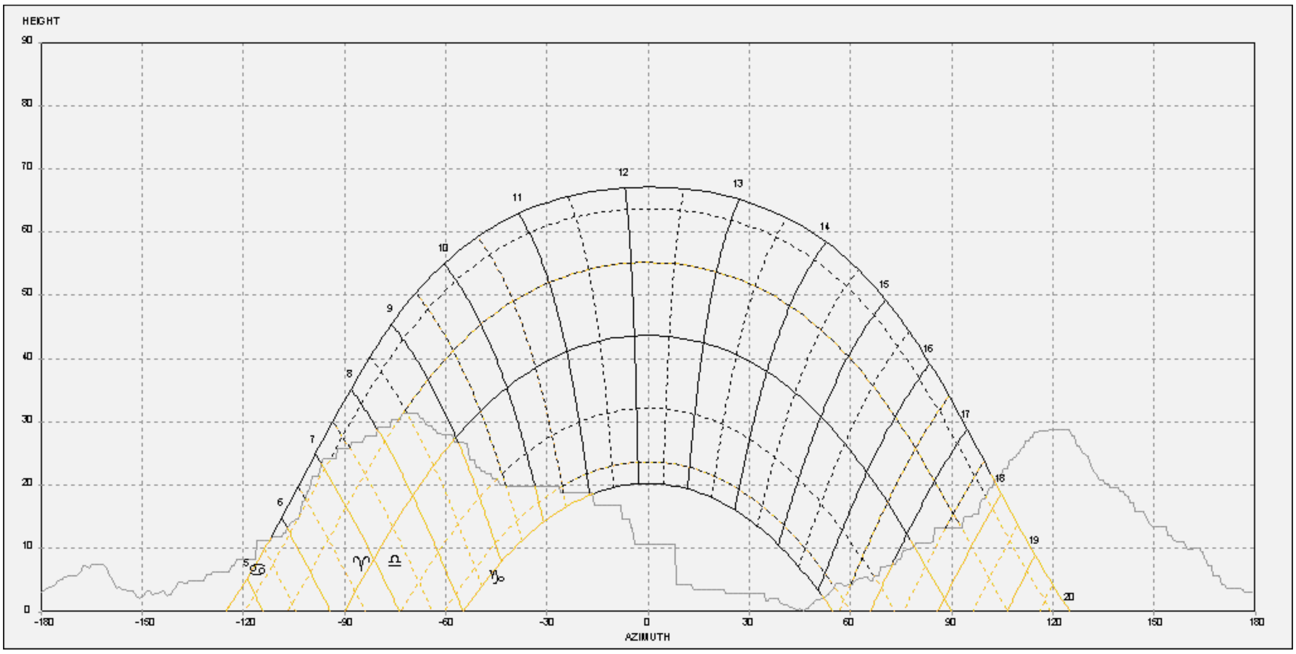

Fig. 11 – Lighting conditions of the dial

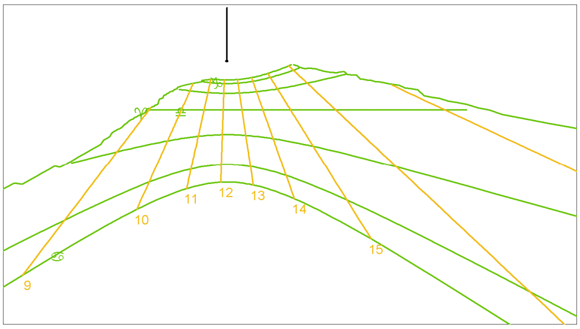

Fig. 12 – True horizon lines on a vertical dial

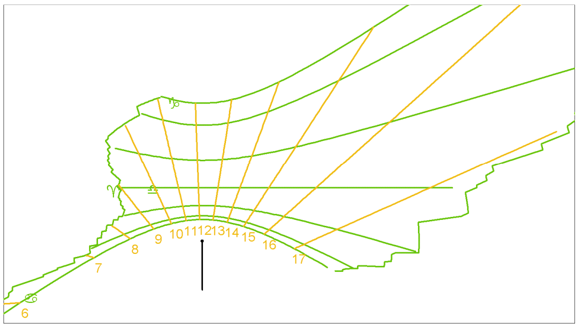

Fig. 13 – True horizon lines on a horizontal dial

The profile of the horizon can also be used to draw the line of the true horizon on the dial in place of the usual theoretical horizontal line (fig. 12).
This true horizon line can even be drawn on horizontal dials (fig. 13) where the theoretical horizon line is at an infinite distance from the dial.

These last two graphs suggest the possibility to draw the lines of the hours that are left before the true sunset (a sort of Italic lines referred to the true instead of the theoretical sunset).
This can be obtained with OS as shown in fig. 14 for a vertical dial and in fig. 15 for an azimuth dial.

Finally the program takes into account the profile of the horizon when in simulation mode, evaluating at each instant the position of the sun with respect to the profile of the horizon.

Conclusion

The knowledge of the profile of the horizon can be used in Orologi Solari to evaluate the lighting conditions of the dial together with all the other influencing parameters:

- latitude
- declination and inclination of the dial
- roofs or balconies
- buildings.

When an experimental measurement is not available, OS can estimate the profile of the horizon starting from the DEM dataset that is available on the web.

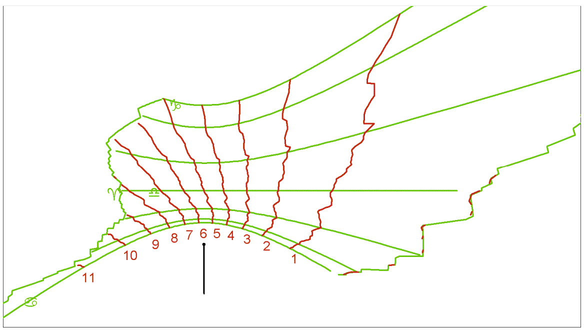

Fig. 14 – Hours left to true sunset on a vertical dial

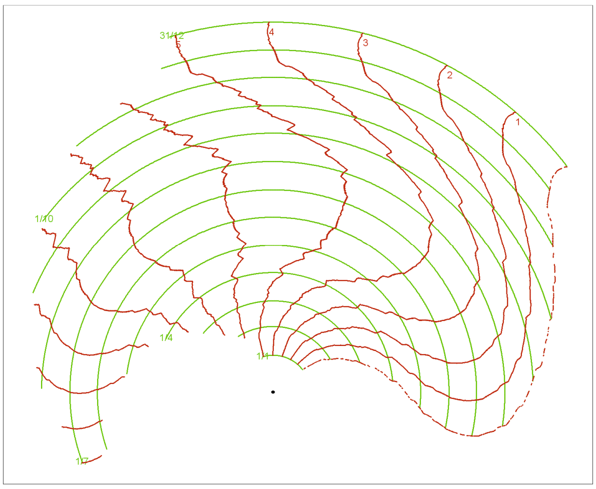

Fig. 15 – Hours left to true sunset on an azimuth dial

Formulae

Points 1 and 2 in the spherical triangle shown in fig. 16 are the observer point and a point of the horizon. The azimuth *A* of point 2 on the horizon as seen from point 1 can be computed as:

$$tan(A) = sin(\lambda_1-\lambda_2) / [sin(\Phi_1) cos(\lambda_1-\lambda_2) - cos(\Phi_1) tan(\Phi_2)]$$

and the angular distance θ between the two points is:

$$sin(\theta) = sin(\lambda_1-\lambda_2) cos(\Phi_2) / sin(A)$$

To compute the height *e* consider fig. 17.
Point 1 at height h_1 is the observer point.
Point 2 is on the horizon at a distance *d* (distance *D* on a straight line) and height h_2.

$$D=sqrt((R+h_1)^2+(R+h_2)^2-2*(R+h_1)*(R+h_2)*cos(\theta))$$

$$cos(e) = (R+h_2) sin(\theta) / D$$

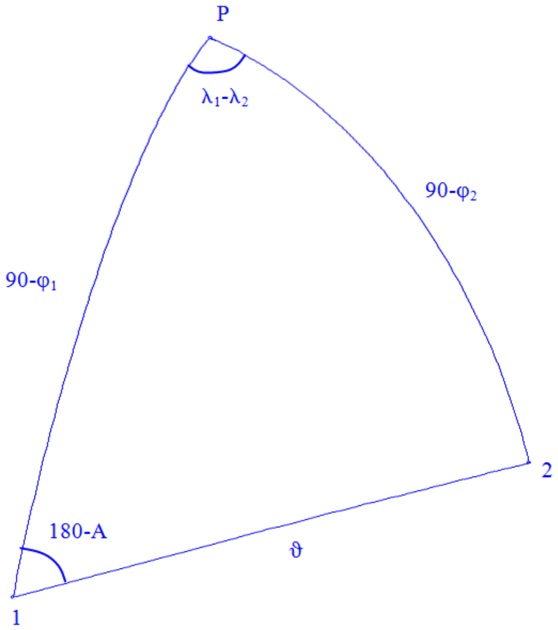

Fig. 16

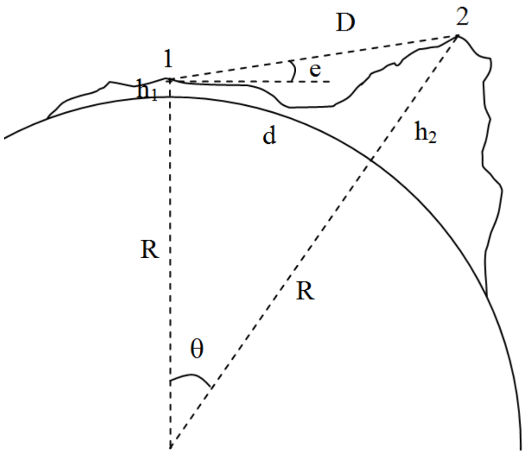

Fig. 17

References

[1] CASALEGNO G. , "Methods And Tools For Sundial Siting", The Compendium 17-1, March 2010

[2] GHIA L., TASSELLI T. , "Orologio Panoramico", XV Seminario Nazionale di Gnomonica, Monclassico 2008

[3] ANSELMI R. , "La linea di occultamento del sole", Gnomonica Italiana n. 7, Novembre 2004

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