Methods and tools for sundial siting

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The choice of the best position on a wall for the design of a sundial is discussed. Different approaches are provided and several examples are shown. The tools offered by the freeware program "Orologi Solari[1]" (OS) are also explained.

Introduction

Design is only a part in the overall activity of sundial manufacturing, maybe not the most difficult.
Have you ever been asked to draw a dial on a north facing wall ? Immediately below a roof or a balcony ? You could say that these are situations that are obviously to be avoided but this is not always the case: because your customer does not believe to your explanations, or maybe because that is the only available place for that house.
You have so to be prepared to perform a deeper analysis and to present rigorous but easy to understand results on the lighting conditions for that wall.

Actually there is no big difficulty in computing the lighting conditions of a planar surface and the shadow that an obstacle like a roof could cast on the dial. The real problem is to perform these computations for several dates in the year and to present the results in a compact but meaningful way.

A vertical dial shows the time only when the sun can light the wall.
This happens when:
- sun is over the horizon
- sun is in front of the wall
- no obstacle is present between the sun and the dial.

The following paragraphs analyze these three conditions showing how OS can help for this activity.

Although the following explanations mainly deal with vertical dials, all the results can be extended to general inclined declining dials.

Sun over the horizon

The height h and the azimuth a of the sun can be computed with the well known formulae :

$$h = \arcsin(\sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\cos(H)) \quad (1)$$
$$a = \arctan(\sin(H) / (\sin(\varphi)\cos(H) - \cos(\varphi)\tan(\delta))) \quad (2)$$

that clearly depend on the latitude φ, the sun declination δ and the time angle H.

At sunrise and sunset the sun height is null, and the corresponding time angle H can be computed as

$$H = \pm\arccos(-\tan(\varphi)\tan(\delta)) \quad (3)$$

[1] "Orologi Solari" can be downloaded from the author's site http://digilander.libero.it/orologi.solari

and the dial is lighted for the time included between these two values.

For a dial on a wall, the latitude φ is fixed and the sun declination δ depends on the date. Different values for sunrise and sunset have so to be computed for each different date. As an example the graph in Fig. 1 shows the time of sunrise and sunset during the year for a place at latitude 45.4 North.

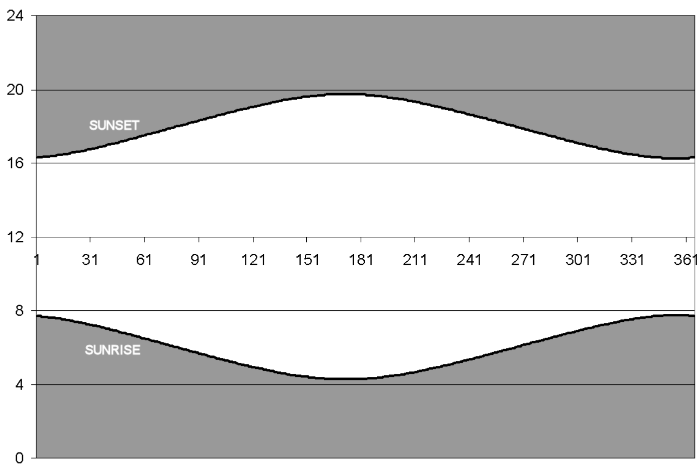

Fig. 1 – Time of sunrise and sunset during the year

OS program makes use of a different method to show this information (Fig. 2): here the position of the sun for each zodiac date and for each hour in the day is shown on an azimuth / height diagram.

The time of sunrise and sunset can be read from the hour lines where they intercept the horizontal axes (height = 0). In this case for example sunrise is at about 4:20 in summer and 7:40 in winter, sunset is at about 19:40 in summer and 16:20 in winter.

The advantage of this kind of graph will become evident going on with this analysis.

Sun in front of the wall

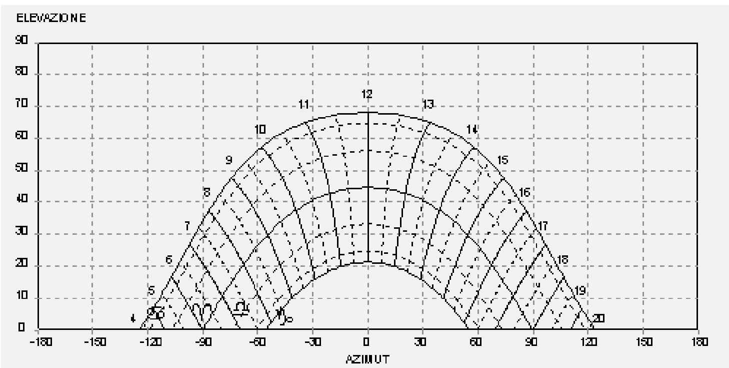

Fig. 2 – OS diagram with sun paths in the sky

In order to evaluate the position of the sun with respect to the vertical wall, the sun azimuth *a* has to be computed. If *d* is the declination of the wall, then the condition "sun in front of the wall" is

$$| a - d | < 90° \qquad (4)$$

In the general case of a declining/inclining wall (inclination *i* is here defined between 0 and 180 degrees and is 90° for a vertical wall) this condition can be expressed in vector notation as

$$\underline{w} \cdot \underline{s} > 0 \qquad (5)$$

where $\underline{w}$ is the unitary vector normal to the wall and $\underline{s}$ is a unitary vector directed toward the sun :

$$\underline{w} = sin(i)sin(d)\ \underline{x} + sin(i)cos(d)\ \underline{y} + cos(i)\ \underline{z} \qquad (6)$$
$$\underline{s} = cos(h)sin(a)\ \underline{x} + cos(h)cos(a)\ \underline{y} + sin(h)\ \underline{z} \qquad (7)$$

$$\underline{w} \cdot \underline{s} = sin(i)sin(d)cos(h)sin(a) + sin(i)cos(d)cos(h)cos(a) + cos(i)\ sin(h) > 0 \qquad (8)$$

that becomes :

$$\sin(d)\cos(h)\sin(a) + \cos(d)\cos(h)\cos(a) > 0 \quad \text{for vertical declining dials } (i=90) \quad (9)$$

$$\cos(h)\cos(a) > 0 \quad \text{for vertical dials facing south } (d=0,\ i=90,\ a=0) \quad (10)$$

$$\sin(h) > 0 \quad \text{for horizontal dials } (i=0) \quad (11)$$

Here the computation of the hours when the dial is lighted becomes slightly more complex: equation 8 (or 9, 10, 11) has to be solved for the hour angle *H*.
Fig. 3 shows again the same graph of Fig. 1 where the time when the sun comes in front of the wall or leaves the wall is also plotted together with sunrise and sunset time (latitude is again 45.4 N, declination 27.6 E).

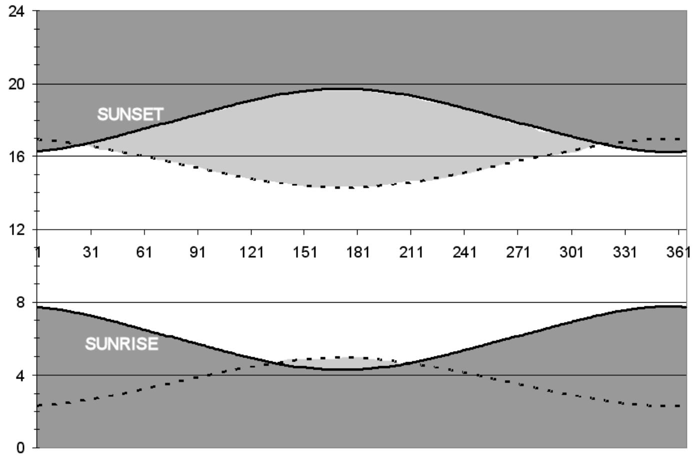

Fig. 3 – Period of time of lighting of the wall in the year

The program OS can help again with the azimuth/height diagram where it puts into evidence the lines corresponding to an illuminated wall with respect to the lines corresponding to the sun behind the wall (see Fig. 4 where black lines correspond to illuminated dial).

Obstacles between sun and dial

If an obstacle like a balcony or a roof is present in the same wall where the dial will be placed, then starting again from (1) and (2) and with some trigonometric calculations, the position of the obstacle shadow for whichever date and hour can be computed.

However the problem here is that it is not possible to write down a general equation that could give as a result the time period when the obstacles obscure the dial and it is so necessary to compute the position of the shadow for all the obstacle corners for each date and time and to determine for each case if the dial is obscured or not.

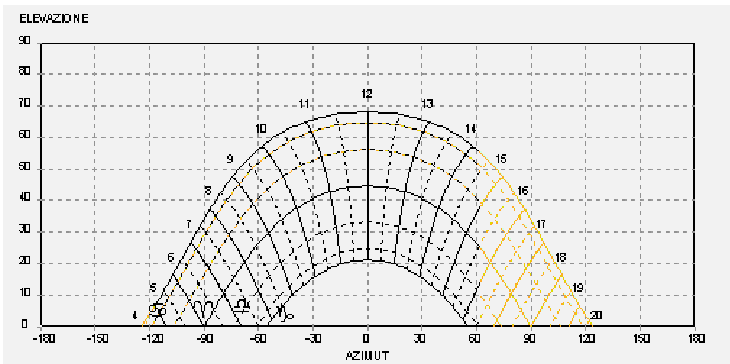

Fig. 4 – OS diagram for a vertical declining dial

This implies to prepare for each single case a spreadsheet or a program and to perform a huge number of computations. Usually some approximation of the problem is adopted to reduce the effort.

One more problem is to present the results in a synthetic and effective way.

A graphical approach to this problem has been suggested in the past [1].

Each element of the obscuring object can be considered as the nodus of a style. Therefore if the corresponding dial is designed and drawn then each point of the dial lines corresponds to the position of the shadow of that element on that date and that time.
This virtual dial can so show the extension of the shadow on the wall for whichever date and time. Of course one element only at a time can be considered. If needed this operation has to be repeated several times, once for each relevant obscuring element.

In Fig. 5 OS has been used to show an example for this method. The wall is the same as in previous examples and one point of the roof is here considered as the nodus of the virtual dial that is plotted on the picture of the wall.
It can be seen that this roof element will obscure the area between the two windows approximately in the time period between 5:45 and 7 AM in winter time only.

This graphical method is simple and fast however it needs to be repeated several times and results cannot be shown together in a single picture.

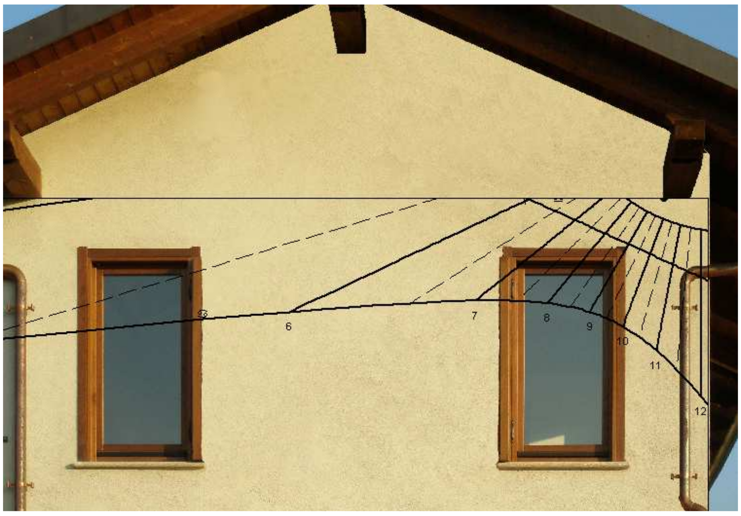

Fig. 5 – Graphical method for roof shadow

With OS the effect of obstacles can be easily added to graphs like those shown in Fig. 2 and Fig. 4.
First of all the obstacle is defined by means of up to 5 corners and for each of them the position on the wall and the prominence length are given (Fig. 6). This operation can be easily done graphically by working directly on the picture of the wall.

For each date / hour combination the position of the shadow of all the corners is computed and then the result is checked against the position of the center (position of the orthostyle) of the dial. The corresponding point in the graph is then colored black for lighted condition and light orange when the shadow obscures the dial center.

In Fig. 7 a dial is positioned in the most natural position on the wall that is between the two windows.
Taking into account the declination of the wall and the effect of the roof previously defined, Fig. 8 shows the overall lighting conditions for the dial.

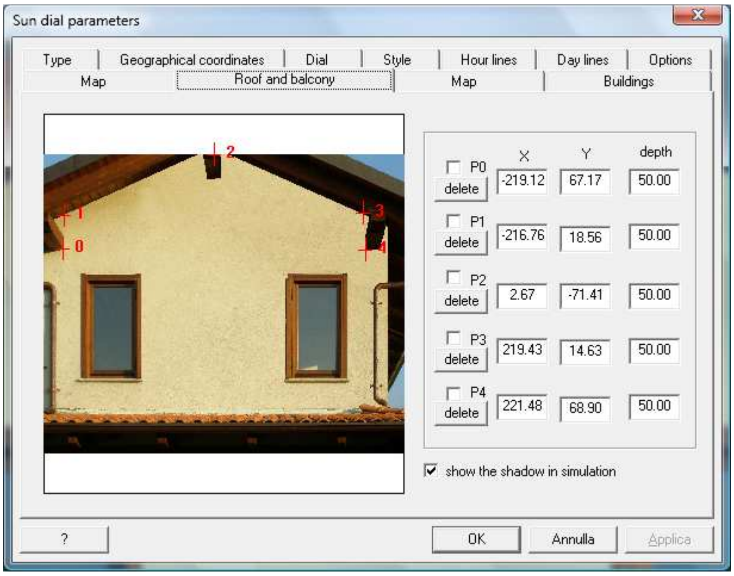

Fig. 6 – Definition of roof elements in OS

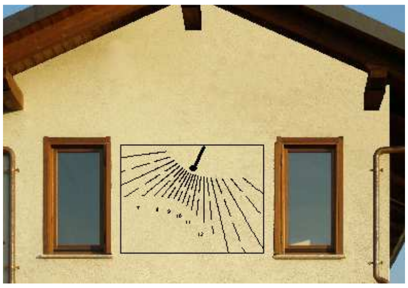

Fig. 7 – 1st dial position

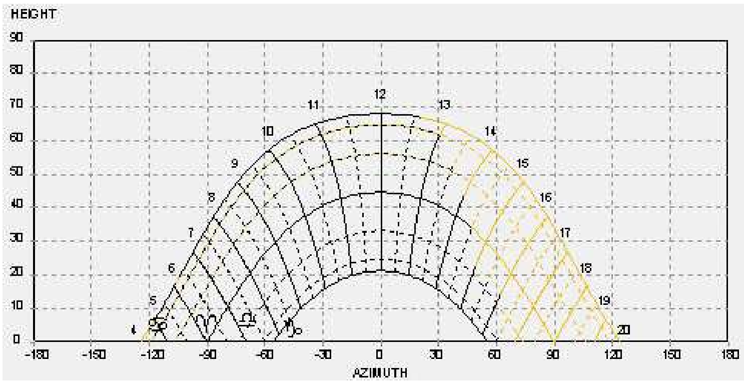

Fig. 8 – 1st dial lighting conditions

Fig. 9 – 2nd dial position

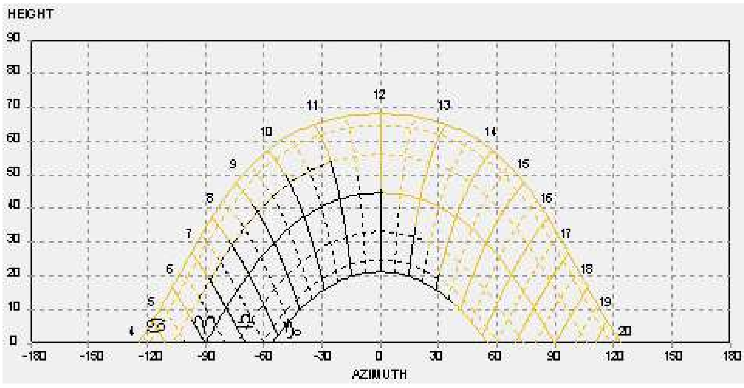

Fig. 10 – 2nd dial lighting conditions

It can be seen that, in addition to the effect of the declination of the wall already taken into account in Fig. 4, there is now a large part of the graph on the summer side that shows how the dial is obscured by the shadow of the roof. At the summer solstice the dial can work until about 12:35 (it was 14:40 in Fig. 4).

If the dial is now moved just below the roof (Fig. 9), obviously the effect of the roof becomes really huge as shown in Fig. 10: the dial will not work for the entire summer season.

The graphs that have been shown until now are an effective way to evaluate the position where the dial should be installed. However let's go back to the question asked in the introduction: "Have you ever been asked to draw a dial immediately below a roof or a balcony ?". If you show an azimuth / height diagram to your customer there is a good probability that he will not be able to understand it.

Fig. 11 – OS simulation of dial and roof

The simulation of how the dial will work in that position would be more easy to understand. Fig. 11 shows three snapshots of the OS simulation for the dial and the roof.

This kind of images do not contain all the information as in the above graphs but they are really effective in explaining the situation to everyone.

Buildings are another common type of obstacles for dials and they also can be managed by OS.

Fig. 12 shows how the position and height of nearby buildings can be introduced.

The effects of the obstacle can again be shown both with the azimuth / height diagram or by means of the simulation of the shadow for whichever time and date in the year (see an example in Fig. 13).

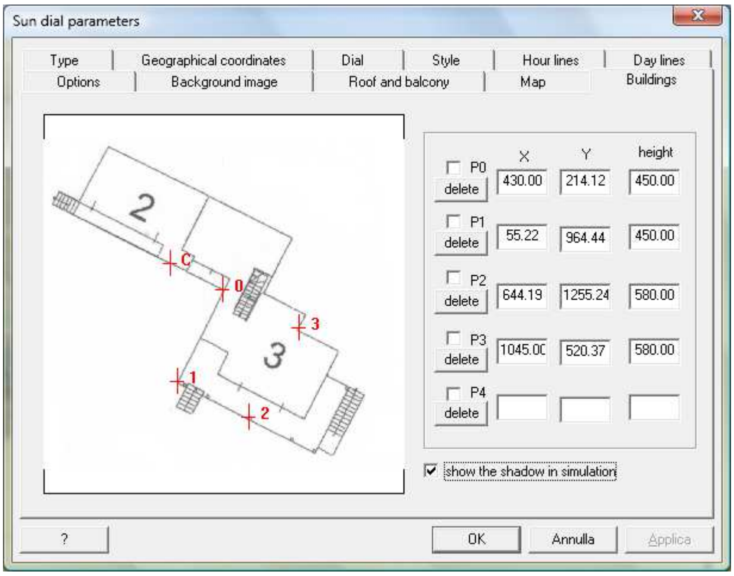

Fig. 12 – Definition of building position and height

Conclusions

The analysis of the best position for a new dial is often neglected mainly because of the large amount of computation that is needed.

OS offers a fast and easy way to compute the lighting conditions of a wall, taking into account roofs, balconies and nearby buildings, in order to present to the customer different solutions for the position of the new sundial.

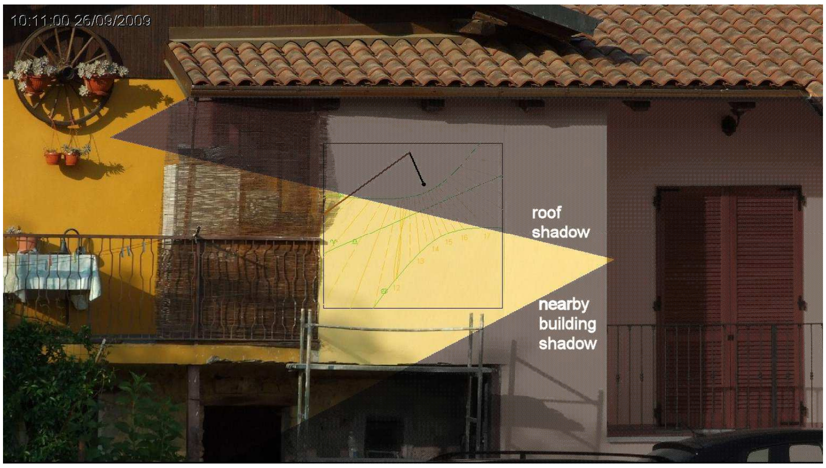

Fig. 13 – OS simulation of dial, roof and nearby building

References

[1] ANSELMI R., *Illuminazione di una parete verticale sovrastante un tetto sporgente*, Gnomonica Italiana n. 14, February 2008

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